NOTATION

$T'(x', y', z', t')$ is the temperature field, x', y', z' are space coordinates; t' is the action time; a, λ are thermal diffusivity and thermal conductivity coefficients; A is the absorptivity; q_0 is the radiation flux density at the center of the heating spot; T_0 is the initial temperature; α' is the coefficient of volume absorption in the Bouger law; and k is the coefficient of concentration in the Gauss law.

LITERATURE CITED

1. N. N. Rykalin, A. A. Uglov, I. V. Zuev, and A. N. Kokora, Laser and Electron Beam Treatment of Materials. Handbook [in Russian], Moscow (1985).
2. N. N. Rykalin, I. V. Zuev, and A. A. Uglov, Principles of Electron Beam Treatment of Materials [in Russian], Moscow (1978).
3. N. N. Rykalin and A. A. Uglov, Fiz. Khim. Obrab. Mater., No. 5, 11-14 (1967).

ALGORITHM FOR THE SOLUTION OF THE PROBLEMS OF BODY HEATING DURING RADIATION HEAT TRANSFER

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UDC 621.1.018

An algorithm is obtained that permits computation of the optimal metal heating mode in a furnace with minimum fuel consumption during radiation heat transfer.

Metal heating modes assuring minimal fuel consumption during heating while taking account of the predominant influence of the fraction of radiation in the welding zone are examined in [1, 2]. A solution is possible for the problem of selecting the optimal modes on the basis of the method of mainline optimization. To do this the results of [3] are utilized, where a theorem has been presented about the decomposition of the original optimal control problem into three sub-problems: periodic optimization and two auxiliary problems of matching the boundary conditions (problems 1 and 2).

Let the metal heating process be described by the differential equation

$$\frac{dT}{dt} = \frac{T_r^4 - T^4}{\mu} \tag{1}$$

with the boundary conditions

$$T(0) = T_0, \quad T(t_k) = T_k. \tag{2}$$

The functional of process quality characterizing the fuel consumption in the furnace has the form [1]

$$B = \int_0^{t_k} \left(M_0 \frac{T_r^4 - T^4}{T_p - T_r} + M_x \right) dt, \quad (M_0 > 0). \tag{3}$$

The following constraints are imposed

$$T_k < A_2, \quad A_1 \leqslant T_r \leqslant A_2, \tag{4}$$

$$\frac{dT}{dt} \geqslant 0, \tag{5}$$

$$t_k \geqslant t_{\min}, \tag{6}$$

Beloruss Polytechnic Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 59, No. 5, pp. 849-853, November, 1990. Original article submitted September 1, 1989.

0022-0841/90/5905-1471$12.50

where

$$t_{\min} = \frac{\mu}{4A_2^3}\left\{\ln\frac{A_2+T_k}{A_2-T_k} + 2\operatorname{arctg}\frac{T_k}{A_2} - \ln\frac{A_2+T_0}{A_2-T_0} - 2\operatorname{arctg}\frac{{}_0}{A_2}\right\}.$$

The constraints (4)-(6) have a specific physical meaning. For instance, the constraint (6) indicates that the total metal heating time in the furnace to a temperature T_k should not be less than the minimal metal heating time to the temperature T_k.

The optimal control problem is to find that function $T_\Gamma(t)$ $(0 \le t \le t_k)$ which would assure a minimum of the functional (3) in solutions of (1) with the boundary conditions (2) and the constraints (4)-(6). We use the results in [3] to solve the formulated problem.

Since the constraint (5) is satisfied, the solution of the periodic optimization problem [3] is written in the form $T^p(t) \equiv T_s$, $T^p{}_\Gamma \equiv T_s$, $t \ge 0$, where T_s is a certain constant; $A_1 \le T_s \le A_2$. For definiteness, let us assume that $T_s = A_1$. We write the problem 1 [3] in the form

$$\frac{dT}{dt} = \frac{T_\Gamma^4 - T^4}{\mu}, \quad T(0) = T_0, \quad T(t_1) = T^p(t_1), \tag{7}$$

where t_1 is the time of arrival of the solution (7) in the optimal periodic mode,

$$\int_0^{t_1}\left(M_0\frac{T_\Gamma^4 - T^4}{T_p - T_\Gamma}\right)dt \to \min_{A_1 \le T_\Gamma \le A_2,\, t_k}.$$

Let us note that the auxiliary problem 1 is an optimal control problem in the Lagrange form with an indefinite process duration t_1; consequently, we have from the conditions of the maximum principle [4] for an optimal process

$$\max_{A_1 \le T_\Gamma \le A_2} H^1(T^1, T_\Gamma, \psi^0, \psi^1) = H^1(T^1, T_\Gamma^1, \psi^0, \psi^1) = 0, \tag{8}$$

where

$$H^1(T, T_\Gamma, \psi^0, \psi^1) = \psi^0 M_0\frac{T_\Gamma^4 - T^4}{T_p - T_\Gamma} + \psi^1\frac{T_\Gamma^4 - T^4}{\mu}.$$

Since the function H^1 is continuous in T_Γ for fixed values of T, ψ^0, ψ^1, it takes on its maximal value in the domain $A_1 \le T_\Gamma \le A_2$, which is achieved either at the boundary points ($T_\Gamma = A_1$, $T_\Gamma = A_2$) or within this domain.

Let us assume that the maximum of the function H^1 is achieved for $A_1 < T_\Gamma < A_2$, then necessarily we obtain

$$\frac{\partial H^1}{\partial T_\Gamma} = 0. \tag{9}$$

It can be shown that the Pontryagin function H^1 for the problem 1 is regular ($\psi^0 \ne 0$), and

$$H^1(T, T_\Gamma, -1, \psi^1) = -M_0\frac{T_\Gamma^4 - T^4}{T_p - T_\Gamma} + \psi^1\frac{T_\Gamma^4 - T^4}{\mu}.$$

On the basis of (9) we write

$$\psi^1 = M_0\mu\,\frac{4T_\Gamma^3(T_p - T_\Gamma) + (T_\Gamma^4 - T^4)}{4T_\Gamma^3(T_p - T_\Gamma)^2}. \tag{10}$$

We obtain $T_\Gamma = T$ from (8) and (10), which is impossible for problem 1. Therefore, the maximum in (8) is achieved for $T_\Gamma = A_2$ or $T_\Gamma = A_1$.

Since $\psi^1(t)$ is continuous in t [4], we obtain from (8) that $T_\Gamma^1(t)$ takes on either the value A_1 or A_2.

On the basis of (7) we conclude that $T_\Gamma^1(t) \equiv A_2$, $0 \le t \le t_1$. Consequently, the time t_1 satisfying condition (7) can be computed from the formula [5]

$$t_1 = \frac{\mu}{4A_2^3}\left\{\ln\frac{A_2+A_1}{A_2-A_1} + 2\,\mathrm{arctg}\,\frac{A_1}{A_2} - \ln\frac{A_2+T_0}{A_2-T_0} - 2\,\mathrm{arctg}\,\frac{T_0}{A_2}\right\}. \tag{11}$$

Therefore, the optimal control and the unknown time t_1 (11) have been found, which after having been substituted into (7) afford the possibility of determining the optimal solution $T^1(t)$ of problem 1.

Problem 2 can be solved in a similar manner [3]

$$\frac{dT}{dt} = \frac{T_\Gamma^4 - T^4}{\mu},$$

$$T(t_2) = T^p(t_2), \quad T(t_k) = T_k,$$

$$\int_{t_2}^{t_k}\left(M_0\frac{T_\Gamma^4 - T^4}{T_p - T_\Gamma}\right)dt \to \min_{A_1 \le T_\Gamma \le A_2,\, t_2}.$$

The optimal control for problem 2 and the time t_2 have the form

$$T_\Gamma^2(t) \equiv A_2, \quad t_2 \le t \le t_k,$$

$$t_2 = t_k - \frac{\mu}{4A_2^3}\left\{\ln\frac{A_2+T_k}{A_2-T_k} + 2\,\mathrm{arctg}\,\frac{T_k}{A_2} - \ln\frac{A_2+A_1}{A_2-A_2} - 2\,\mathrm{arctg}\,\frac{A_1}{A_2}\right\}.$$

Let $T^2(t)$ denote the optimal solution of problem 2.

According to [3] the extremal for the initial optimal control problem is the trajectory

$$T(t) = \begin{cases} T^1(t), & 0 \le t \le t_1, \\ T^p(t), & t_1 \le t \le t_2, \\ T^2(t), & t_2 \le t \le t_k \end{cases} \tag{12}$$

Let us show that T(t) is the optimal trajectory.

We represent the functional (3) in the form

$$B = \int_0^{t_k}\left(M_0\frac{\mu\dot{T}}{T_p - T_\Gamma} + M_x\right)dt.$$

We have for the mode (12)

$$B = \frac{M_0\mu}{T_p - A_2}\int_0^{t_1}\dot{T}dt + \frac{M_0\mu}{T_p - A_1}\int_{t_1}^{t_2}\dot{T}dt + \frac{M_0\mu}{T_p - A_2}\int_{t_2}^{t_k}\dot{T}dt + M_x t_k =$$
$$= \frac{M_0\mu}{T_p - A_2}(T_k - T_0) + M_x t_k.$$

On the other hand, for any solution of (1), (2), (5) we have

$$B \ge \int_0^{t_k}\frac{M_0\mu\dot{T}}{T_p - A_2}dt + M_x t_k = \frac{M_0\mu}{T_p - A_2}(T_k - T_0) + M_x t_k.$$

consequently, the function (12) is the optimal solution of the problem formulated.

Let us note that it is not unique. By selecting $T_s = T_k$ we also obtain an optimal solution of the problem.

The following fact is an interesting feature of the optimal control problem under consideration. Let the productivity of a rolling mill drop for some reason, then this will result in an increase in the time the ingot is in the furnace t_k. However, the nature of the behavior of the optimal solutions does not change: as the fixed time of the heating process grows, the optimal solutions tend to trajectories that are obtained because of solving the periodic optimization problem. The greater the heating time, the greater the times the optimal solutions are on these trajectories independently of the boundary conditions. These trajectories are called mainlines [6], and problems 1 and 2 are problems of matching the boundary conditions to the mainline.

Therefore, an algorithm has been obtained for minimization of the fuel consumption during heating a metal in a furnace that permits obtaining optimal solutions independently of the productivity of the tool.

NOTATION

t is the time, sec; T is the metal temperature, K; T_Γ is the smoke gas temperature, K; T_0 is the initial metal temperature, K; μ is a time constant; t_k is the given duration of the heating process, sec; T_p is the adiabatic temperature of fuel ignition, K; M_x is the furnace no-load power, W; A_1, A_2 are the lower and upper bounds of smoke gas temperature variation, K.

LITERATURE CITED

1. E. M. Gol'dfarb and V. S. Ibryaev, Izv. Vyssh. Uchebn. Zaved. Chern. Metall., No. 1, 115-117 (1973).
2. A. R. Kavaderov and Yu. A. Samoilovich, Inzh.-Fiz. Zh., No. 7, 110-113 (1959).
3. V. B. Kovalevskii, V. I. Panasyuk, and O. Yu. Sedlyako, Izv. Vyssh. Uchebn. Zaved. Energ. (1989). Dep. No. 3976-B89 in VINITI, June 14, 1989.
4. L. S. Pontryagin, et al. Mathematical Theory of Optimal Processes [in Russian], Moscow (1976).
5. G. P. Ivantsov, Heating of Metals [in Russian], Moscow (1948).
6. A. I. Panasyuk and V. I. Panasyuk, Asymptotic Mainline Optimization of Control Systems [in Russian], Minsk (1986).